**LECTURE 5**

**ANALITICAL MATHEMATICS**

**The analytical geometry’s simplest tasks**

 **1. The Distance between points.** The **distance** *d* between the points  and  is

 units.

**2. Division the segment in a given ratio**. Consider a straight line passing through the points  and . Given a point *М(х,у)* on the line *АВ.*  Let the point divides the segment in the ratio *АМ:МВ=.*In this case, the coordinates of the point *М(х,у)* are:

, .

Consider the particular case. The point half-way between two points is called their midpoint, i.e. *=1:1=1*. It is calculated as follows. The **midpoint** of and  is

, .

**3. Square of the triangle.** In the plane, square of a triangle vertices of which at the points , , is calculated by the following formula:

.

**THE EQUATION OF A CURVE ON A PLANE**

Suppose given an arbitrary curve in the Cartesian plane. Consider the equation

B

 A















***F(x, y)=0*** (1)

describing the relation between *x* and *y*.

For example:

*x - 5y – 2 = 0, x2+y2 – 16 = 0*.

The equation of a curve is the basic concept in analytic geometry.

**Def.***If all points of the curve satisfy the equation (1), and no other point does not, then (1) is* ***the equation of the curve.***

The coordinates of  in the figure satisfy the equation

,

because it belongs to the curve, but due to the fact does not belong the curve, its coordinates cannot satisfy (1):

.

The concept of the equation of the curve makes it possible to calculate the geometric problems algebraically. For example, search for points of intersection of the curves given by the equations *x-y-2=0, x2+y2-16=0*. It needs to solve a system of these equations, ie, solve algebraic problems.

**DIFFERENT EQUATIONS OF A CURVE**

 The line in a coordinate plane crosses the y-axis at the point *В(0;b)*, and the angle between the line and the x-axis is equal to (0<<) (Fig.1). М(х,у) is an arbitrary point on the line. Tangent of the angle  is sought in the triangle ВМК.















Figure 1

 (1)

Designation **, called **the slope of the line**. Thus:

.

Hence we find *у,* the following equation:

*y=kx+b* (2)

All points of the line satisfy this equation, and no other point does not. This equation is **given by the slope-intercept form**.

Consider particular cases

1. Let *b=0*. Then *y=kx,* line passes through the origin (Fig.2)

**2**. Let , then  and *y=b.* Thus line is parallel to the x-axis (Fig.3). By the way, the equation of the x-axis is *y=0*.



*x=a*



























Figure 2 Figure 3 Figure 4

### **3**. If , then  undefined, line is perpendicular to the x-axis. the line crosses the x-axis at a, then its equation is х=а (Fig.4). The equation of the y-axis is x=0.

###  We have the following theorem.

 **Theorem*.* Any line in the Cartesian coordinate system can be given by equation of the first order**

***Ах+Ву+С=0* (3)**

**and, conversely, the equation (3) in the Cartesian coordinate system defines direct (if all the coefficients are not simultaneously zero).**

 Usually (3) is called the general equation of line.

**Equation of a line in a given direction and passing through a given point.** Sometimes, it is necessary to write the equation of the line, knowing a point ****belonging to it, and its slope *k*. (Fig.5).



























Figure 5 Figure 6

Let us write the equation in the form (2), *y=kx+b*, here *b*  is temporarily unavailable. Point belongs to the line, so its coordinates satisfy the equation of the last one, like this: *y*1*=kx*1*+b.* Hence we can calculate the unknown *b*, *b = y*1 - *kx*1.  Substitute the found value in the equation, and obtain the equation written for a given direction and the point:

*y =k(x – x*1)*+ y*1 (4)

If *k* - arbitrary, through  passes a bundle of rays (Fig.6)

 **The equation of the line passing through 2 points. Consider two points ** and . In order to write the equation of the line *АВ* write the equation a beam of rays that pass through the point *А*:



















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*y =k(x – x*1)*+ y*1.

Line АВ passes through the point too. This means that its coordinates satisfy one of the ray equations: *y*2 *=k(x*2 *– x*1)*+ y*1. Hence calculate the unknown *k*, . If this unknown is substituted in the equation, we get the equation of the line passing through the two points A and B:

 (5)

 **The equation of the segment.** Let the line measured from the origin on the x-axis the segment length is equal to *a*, and on the y-axis the segment length is equal to *b* (Fig.8). We use formula (5) for the points *А(а;0)* and *В(0;b)*. Thus the equation of the line is:















Figure 8



We simplify the expression, and obtain an equation for the segment:

 (6)

 **Angle between two lines.** Given two lines: *y=k*1*x+b*1, *y=k*2*x+b*2. Here , . How to find the angle between these lines? (Fig.9).

As shown in Figure . Hereof













Figure 9



or

 . (7)

Formula (7) determines the angle between the given. The other angle is .

 **Conditions of parallelism and perpendicularity of two lines.** If the lines are parallel, then =0 and *tg*=0. In this case, the formula (7) is as follows: *k*2 – *k*1 = 0. Hence it follows condition of parallelism of two straight:

*k*2  = *k*1 , (8)

ie if the slopes of the lines are equal, then they are parallel, and vice versa.

 If the lines are perpendicular, then , and , . Hence it follows condition of perpendicularity of two straight:

*k*2  = , (9)

ie if slopes are inverse with a negative sign of each other, these lines are perpendicular, and vice versa.











Figure 10

**Distance from a point to a line.**

In a rectangular coordinate system defined line *Ах+Ву+С=0* and point *М(х0,у0)* not in line. (Fig.10).

***Distance from point to a line*** is the length of the perpendicular dropped from the point to this line. *d=MN*. To find the distance: а) find the equation of the line that passes through the point *М(х0,у0)*  and perpendicular to the given line; б) solve the system of equations of the given equation and *MN,* and find the point *N* of intersection of their; в) using the formula for calculating the distance we find the distance *d=MN*. Finally, we obtain the following formula:

 (10)

**Example.** Consider a triangle with vertices *А(1;1), В(7;4), С(4;5).* Let us find:

а) the length of the side AB;

b) equations of the lines AB and AC;

c) the inner corner A;

d) equation of height and the median drawn from vertex С;

e) distance from the vertex С to the line АВ.



**Decision**. а) Using the formula for calculating the length of the segment we find the length of AB:











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b) We find the equation of the line using the following formula:

.

Here ** =** A and  = В: , simplify, and obtain

.

Similarly, an equation for the А: , hence .

c) For the calculation the inner angle use (7). From the previous paragraph we take the slopes of the two lines - AB and AC, , , and substitute in the formula (7):

,

hence .

г) Denote the height as СD. Using the equation *y =k(x – x*1)*+ y*1 to write the equation of CD. If we substitute instead of the coordinates ****, we get the equation a beam of rays passing through the point С: *y =k(x -4*)*+ 5*. Now remember that the line should be perpendicular to AB: . Substitute the coefficient in the equation and we get: *y =-2(x -4*)*+ 5* . So,

*y =-2x+13.*

 To write the equation of the median СЕ, necessary find the coordinates of the middle of the:

, , *Е=(4; 2,5).*

Using the formula to find equation of the line passing through the two points:

,

hence *х=4* is equation of *СЕ.*

д) To find the distance from the vertex С to the line AB, we should write the equation of АВ in following form:

*x - 2y + 1 = 0.*

Then use formula (10). Hence we get:

.**THEORETICAL ISSUES AND CHELLENGES**

* Think of how to divide the segment in a given ratio.
* Calculate the equation of the line with a given slope.
* Write the equation of the line passing through 2 points.
* How can you calculate the distance from a point to a line?
1. Given a triangle with vertices *А(2;-1), В(4;3), С(-2;1)*. Need to find the lengths of the edges and the coordinates of the midpoints of the edges.
2. Given a triangle with vertices *О(0;0), А(8;0), В(0;6).* Find the length of the median ОС and the bisector OD*.*
3. Given a triangle with vertices *А(4;1), В(7;5), С(-4;7).* Find the point of intersection of the medians.
4. Given a triangle with vertices *А(2;0), В(5;3), С(2;6).* Find the square of the triangle.
5. Find equation of a line equidistant from the points *А(0;2)* and *В(4;-2)*. Do following points belong to this line: *С(-1;1), D(1;-1), E(0;-2), F(2;2)*?
6. Find equation of a line equidistant from the point А(2; 1) and a line *x=0*.
7. Find equations of the lines that cross the y-axis, cutting *b=3* and the angle of the x-axis is:

1) 450;

2) 1350.

Draw them.

1. Find the parameters *b* and *k*:

a) *2x-3y=6*; b) *2x+3y=0*; c) *y=-2*; d) .

1. Find the angles between the following lines:

a) *5x-y+7=0, 2x-3y+2=0* ; b) *3x+2y=0, 6x+4y+7=0.*

1. In a rectangular coordinate system draw the point А(-2;5) and the line *2х-у=0*. Write the equation a beam of rays passing through. Among these lines find parallel and perpendicular to the given line.
2. Find the equation of the line passing through the points *А(-1;3)* and *В(4;2)*.
3. Given a triangle with vertices *А(-2;0), В(2;6), С(4;2*), and median ВЕ and height BD of the triangle are given. Write the equations of lines АС, ВЕ and BD.
4. Write the equation of the line passing through the origin, and which has an angle 450 with the line *y=4-2x*.
5. Find the distance from the points *А(4;3), В(2;1)* and *С(1;0)*tothe line *3x + 4y - 10=0*.
6. Businesses spend to produce 300 units of goods 1000000 units of cost. And for 500 units of goods 1600000 units of cost. Define the linear pattern of expenditure of the businesses.
7. The initial price of the copier is 9000$. After 5 years, the rice will be 1000$. Find copier price in 3 years.
8. Initial price of computer is 700$. After two years, the price will be 480$. Find computer price in 5 years.
9. Dependence of the price on the transported distance for the first transport: *y=150 + 50x* (per each km x, y – price of transporting). And for the other transport: *y=250 + 25x*. Which transport and at what distance more advantageous to transport goods?

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1. Given a triangle with vertices *А(-2;0), В(6;6), С(1;-4)*. Find the length of the bisector АЕ.
2. Find the coordinates of the point *M,* if it divides *AB* as follows*:*

*АМ:МВ=-3:2*

And if *А(-2;1), В(3;6).*

1. Prove that the points line *А(1;1), В(-1;7), С(0;4)* lie on a straight line.
2. Given a quadrangle with vertices *А(3;1), В(4;6), С(6;3)*  and *D(5;-2).* Find the square of it*.*
3. The point *М(х,у)* in two times closer to the point *А(-1;1)* than the point *В(-4;4)*. Find the trajectory of *М(х,у)*.
4. Find the geometric arrangement of points equidistant from a point *М(2;2)* and a line *y=0.*
5. Write the equations of the lines with the parameters 1) b=-2, =450; 2) b=-2, =1200 and draw them.
6. Write the equation of a line, parallel and perpendicular to the line *3х+5у=7*, and passing through М(2,-4).
7. Given a triangle with vertices *А(1;-1), В(4;3), С(5;1)*. Find:

а) equations of lines АВ and ВС; b) the inner angle of B;

в) equations of median АЕ and height СD;

1. Prove that the lines *2x-3y=5, 6x-9y=11* are parallel. Find the distance between them.
2. The initial price of the car is12000$. Lifetime is 8 years. Price of past due car is 2000$. How much does it costs after 6 years?